Theorem Proving the Existence of Contradiction Minimum Contradictions Fuzzy Thinking and Physics in Logical Communication

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***Abstract*- The purpose of this paper is to show, by means of a theorem, that the logical communication system, through which every theory is stated, is contradictory; this imposes silence which can be broken through the claim for minimum contradictions. Even though at first this theorem has been proven previously, it constitutes the basic element of this paper. Here is offered a proof without the weaknesses existing in previous publications. This theorem can explain the reason why fuzzy logic is permitted as an inexact way of thinking; it leads to a minimum contradictions physics which implies quantum gravity and which is compatible to fractal geometry. On this basis the laws of nature are the principles of our logical communication with the claim for minimum contradictions included.**

Keywords- Theorem; Logical Communication; Sufficient Reason Principle; Minimum Contradictions; Fuzzy Logic; Fractal Geometry

I. INTRODUCTION

Every theory includes, beyond its particular axioms, the principles of the basic communication system (language) through which it is stated. This system obeys the Aristotelian logic (Classical Logic) [1, 2], the Principle of Sufficient Reason [3, 4] according to which for everything we seek the reason of its power and a hidden axiom which states that there is anterior-posterior everywhere in communication [5]; in fact, the way in which we communicate is not a simultaneous process; one word is put after another etc. We denote by Λ a logic consisting of the Classical Logic denoted as P_I and the Principle of Sufficient Reason regarded as a Complete Provability Principle denoted as P_{II} and defined as:

Complete Provability Principle - P_{II} : "No statement is valid if it cannot be logically proved through a complete set of valid statements different from it."

where a complete set of valid statements-reasons does not require further valid statements beyond it; if there is not a complete set of valid statements-reasons, then there is not a complete and clear proof for a statement validity which is in contrast to the Principle of Sufficient Reason. We may notice that Λ corresponds to the complete Aristotelian Formal Logic i.e. Classical Logic [1,2] plus Principle of Sufficient Reason (*in Greek* : Αρχή του Αποχρώντος Λόγου) [3] at its most severe form where no doubt can exist; it is noted that the Sufficient Reason Principle is widely known as a Leibniz's principle because of his decisive contribution to this subject [4]. This principle expresses the deeper human desire for everything to be clear and explained.

On this basis the following can be proven:

Theorem I: "Any system that includes logic Λ and a statement that is not theorem of logic Λ leads to contradiction."

It can be proven that a statement which is not theorem of Λ is the following:

Anterior-Posterior Axiom: "There is anterior-posterior everywhere in communication."

Thus, because of Theorem I we can state:

Statement I: "Any communication system that includes the logic Λ and the anterior-posterior axiom leads to contradiction."

This implies that the logical communication system is contradictory; this imposes silence which can be broken through the claim for minimum contradictions.

Even though at first, Theorem I has been proven previously [5, 6], it constitutes the basic element of this paper and offers a proof without the weaknesses existing in the previous publications.

Theorem I and Statement I can explain the reason why the use of fuzzy logic is permitted while it is not an exact way of thinking; they lead to a minimum contradictions physics which implies quantum gravity and which is compatible to fractal geometry. Minimum contradictions physics, under certain simplification, is compatible to Newtonian mechanics, QM and relativity theory.

II. THEOREM

The following theorem will be proven as valid:

Theorem I: "Any system that includes logic Λ and a statement that is not theorem of logic Λ leads to contradiction."

A. General

For the purposes of this paper we use a symbolic logic [7] so that logic Λ can be used. Thus, we have:

$$\Lambda \equiv P_I \cdot P_{II} \tag{1}$$

$$P_{II} \equiv P_{IIa} \cdot P_{IIb} \tag{2}$$

$$P_{IIa} : \sim Proof_{\Lambda}(p, p) \tag{3}$$

$$P_{IIb} : p_C \Rightarrow \wp_C \cdot Proof_{\Lambda}(\wp_C, p) \tag{4.1}$$

$$p \Rightarrow \wp \cdot Proof_{\Lambda}(\wp, p) \tag{4.2}$$

where P_I stands for classical logic and P_{II} stands for the *complete provability principle.*

P_{IIa} states that it is not valid that statement- or set of statements- p can prove itself on the basis of logic Λ.

P_{IIb} states that if p is valid then a statement- or *complete* set of statements- $\wp_C$ is valid so that p can be proved by means of $\wp_C$ through logic Λ; a complete set of valid statements does not require further valid statements beyond it.

An immediate consequence of statement (4.1) is statement (4.2) which states that if p is valid then a statement- or set of statements- $\wp$ is valid so that p can be proved by means of $\wp$ through logic Λ; this is related to an intermediate proof, not to the complete one.

P_I i.e. the Classical Logic is the proof language of the present theorem and it is regarded as valid, since otherwise none statement could be stated. In a system including Λ, because of (3) applied for $p \equiv P_I$, P_I is not a priori valid. Therefore the present theorem is valid under the hypothesis that P_I is valid as a language of reference; this does not reduce the value of this theorem since out of classical logic P_I, at a first sight, nothing can be stated i.e. Theorem I cannot be stated and this leads to silence (see Section III). It is noted that silence imposed by logical communication constitutes the basis of the minimum contradictions thinking (see Section III) which constitutes the basic application of the present theorem.

B. Proof

According to P_{II}, Λ is not a priori valid and it needs a proof for its validity so that it can be used.

Because of P_I which is the language of this proof we can write:

$$\Lambda \vee \sim \Lambda \tag{5}$$

which means that either logic Λ is valid or logic Λ is not valid. Therefore, we can look into the following cases:

1) Logic Λ Is Not Valid

It is obvious that if a system includes Λ this system is contradictory, since it must be valid Λ and ($\sim \Lambda$) at the same time.

2) Logic Λ Is Valid

We consider the system:

$$[\,\Pi \equiv \Lambda \cdot p \cdot q \tag{6}$$

If:

$$p \cdot q \equiv p', \tag{7}$$

$$\text{“}\Pi \textit{ is complete”} \tag{8}$$

then, we will have that p, q must be provable through Λ, p, q; since p, q according to P_{IIa} cannot prove themselves. We will have:

$$p \Rightarrow Proof_{\Lambda}(\Lambda, p) \vee Proof_{\Lambda}(q, p) \tag{9}$$

$$q \Rightarrow Proof_{\Lambda}(\Lambda, q) \vee Proof_{\Lambda}(p, q) \tag{10}$$

By hypothesis there is a statement of Π which is not theorem of Λ; let p be this statement. Thus we will have:

$$\sim Proof_{\Lambda}(\Lambda, p) \tag{11}$$

Because of statements (9, 10, 11) we obtain:

$$p \cdot q \Rightarrow Proof_{\Lambda}(\Lambda, q) \cdot Proof_{\Lambda}(q, p) \vee Proof_{\Lambda}(q, p) \cdot Proof_{\Lambda}(p, q) \tag{12}$$

Both terms of right part express impossibility; in fact applying classical logic we have:

$$Proof_{\Lambda}(\Lambda, q) \cdot Proof_{\Lambda}(q, p) \Rightarrow Proof_{\Lambda}(\Lambda, p) \tag{13}$$

i.e. if Λ proves q and q proves p then Λ proves p; this is in contrast with statement (11). Working in the same way we have that:

$$Proof_{\Lambda}(q, p) \cdot Proof_{\Lambda}(p, q) \Rightarrow Proof_{\Lambda}(q, q) \tag{14}$$

which is in contrast with principle P_{IIa}. Thus, because of statements (11, 12, 13, 14) we have that Π leads to contradiction; consequently we can state:

Statement II: “If logic Λ is by hypothesis valid, then any system that includes this logic Λ and a statement that is not a theorem of logic Λ cannot be complete and consistent at the same time.”

As one can notice this statement has basic similarities with Gödel’s theorem [8] and its version according to Rosser [9]; however it does not need the existence of any arbitrary algorithm as Gödel’s theorem does [10].

Because of P_{II} we have that Λ and $p \cdot q \equiv p'$ must be provable through a complete set of valid statements $\wp'_C$ different from them; as was mentioned, Λ is by hypothesis valid. Because of statement (4.1) we obtain:

$$p' \Rightarrow \wp'_C \cdot Proof_{\Lambda}(\wp'_C, p') \tag{15}$$

We consider the system:

$$\Pi_{TOT} \equiv \Lambda \cdot \wp'_C \cdot p' \tag{16}$$

This system includes p' and therefore p which is not theorem of Λ; therefore it obeys the statement II which implies that it cannot be complete and consistent at the same time. Therefore, there does not exist the complete proof $\wp'_C$ for p' validity which implies that:

$$p' \Rightarrow \neg \wp'_C \cdot Proof_{\Lambda}(\wp'_C, p') \tag{17}$$

Because of statements (15, 17) we have contradiction and taking into account Section B.1 we obtain *Theorem I,* regardless of whether Λ is valid or not i.e.:

Theorem I: "Any system that includes logic Λ and a statement that is not theorem of logic Λ leads to contradiction."

III. MINIMUM CONTRADICTIONS THINKING

We name “0” the state before our communication and 1, 2, 3 … the sequent states, e.g. written symbols, of this communication. State “0” corresponds to the non-existence of any communication symbol while state “1” corresponds to the existence of the symbol “1”. We notice that from the non existence of the symbol “1” (e.g. state “0”) we cannot derive logically the existence of the symbol “1” (state “1”). Working in the same way we have that, in general, a “posterior” does not derive logically from its “anterior”. In fact the “ n ” state can correspond to the absence of the “ $n+1$ ” symbol; from the non existence of the symbol “ $n+1$ ” (state “ n ”) we cannot derive logically the existence of the symbol “ $n+1$ ” (state “ $n+1$ ”). Therefore the Anterior-Posterior Axiom is not theorem of Λ . Applying Theorem I we obtain *Statement I i.e.* [5]:

Statement I: "Any communication system that includes the logic Λ *and the anterior-posterior axiom leads to contradiction."*

This implies that the logical system, through which we communicate and every theory is stated, is contradictory; despite this, we do communicate in a way we consider logical *avoiding to make mistakes on purpose*. Since contradictions are never vanished, we try to understand things through minimum possible contradictions. On this basis we can state [5]:

The Claim for Minimum Contradictions: "What includes the minimum possible contradictions is accepted as valid."

According to this claim we obtain a logical and an illogical dimension. In fact, through this claim we try to approach logic (minimum possible contradictions), but at the same time we expect something illogical since the contradictions cannot be vanished.

All axioms mentioned, the claim for minimum contradictions included, constitute the principles of the active logical language; when we speak logically we persist in logic despite of the existing contradictions.

According to what was mentioned, our basic communication system obeys Statement I; however, we notice that Statement I cannot be stated because it is based on the basic communication system which, according to Statement I itself, is contradictory. Therefore we may notice that:

Statement I imposes the silence.

Thus, the claim for minimum contradictions includes the arbitrariness derived from breaking the silence. Thus we can reach a deeper truth through silence; this might facilitate us in a better understanding of notions as free will, faith etc. All these do not derive from a metaphysical point of view but from an analytical approach to the limits of thinking.

On this basis determinism is broken since it cannot coexist with contradictions. Thus, the limit inference of ratio is its rejection and this is compatible with the existence of free will. We can reach the same results through the work prior to Gödel’s and Rosser’s theorems [5, 8, 9]. However, these theorems require an arbitrary hypothesis according to which “there is an algorithm able to prove the true statements” [10] which, until now, has been successfully applied only for the case of the Turing machine [11]. The theorem of the present work overcomes this restriction and it can apply to the human logical communication.

A paradigm of *minimum contradictions thinking* is the *minimum contradictions physics* as it will be explained below.

IV. MINIMUM CONTRADICTIONS PHYSICS

Every theory includes at least the principles of the basic communication system. According to Theorem I, further axioms beyond the ones of basic communication must be avoided since they can cause further contradictions. The systems of axioms we use in physics include the communication system and, therefore, their contradictions are minimized when they are reduced to the communication system itself.

At first sight, for a minimum contradictions physics we can make the following statement [5, 6]:

Statement II: In a minimum contradictions physics everything is described in anterior–posterior and in extension in space-time terms.

Since there is nothing else than space-time, because of differences at various points of matter systems, time must have different flow rates at different points; therefore it should be regarded as a 4^{th} dimension which implies Lorentz’ transformations and in extension a relativistic theory [5].

At second sight, applying the claim of the minimum contradictions, we conclude that matter-space-time has logical and contradictory behaviour at the same time; this can be valid when space time exists and does not exist at the same time (illogical behaviour) while it implies the existence of at least one logical statement (logical behaviour); this is compatible to the concept that space-time is stochastic and it has a probability to exist in an infinitesimal area around a point $(\mathbf{r},t)$ of a Hypothetical Measuring Field (HMF) [5]. When the probability integral equals to 1 the following statement is valid: *“there is space-time”* which is a logical statement. Thus, we can state the following:

Statement III: Minimum contradictions physics can be described by stochastic space-time.

Statement III has sense if there are various kinds of space-time corresponding to the various forms of matter. Thus we can use signs (± 1) for (g) -mass and ($\pm i$) for (em)-charge space-time.

The stochastic space-time derives from the distribution of the properties of a flat relativistic space-time according to the probability density $P(\mathbf{r},t)$ of Schrödinger’s relativistic equation which can derive, by Fourier analysis, from the principles of the logical communication [5].

The minimum contradictions physics is a stochastic matter-space-time QM implying quantum gravity and under certain simplifications the Newtonian mechanics and the relativity theory [5]; it implies the self-similarity property, in a unified quantum space time field, which constitutes the basic property of fractal geometry [5,12] and the existence of space time operators [5].

On this basis, the laws of Nature are the principles of our logical communication i.e. the principles of logical language the claim for minimum contradictions included.

V. FUZZY LOGIC, MINIMUM CONTRADICTIONS AND FRACTALS

Fuzzy Logic is an inexact way of thinking which however has a wide range of applications in physical systems [2]; thus, the question is raised of what permits the inexact way of thinking.

According to this paper and more specifically because of Theorem I and Statement I, the most consistent attitude is silence. Thus, any real communication system can derive by breaking the silence and taking into account the existence of contradictions. Therefore any real communication system constitutes an inexact way of thinking, i.e. it is a kind of fuzzy logic; it is noted that many fuzzy systems have been proposed [13].

According to this paper, one kind of fuzzy logic is the minimum contradictions thinking which takes into account the contradictions existence through the maximum use of logic (minimum contradictions); this kind of logic i.e. this fuzzy system implies the minimum contradictions physics which, as was mentioned, is compatible to modern physics and implies the self-similarity property which characterizes the fractal geometry [12].

On this basis the minimum contradictions thinking is close to fuzzy logic and fractal geometry which have been widely applied.

REFERENCES

[1] Aristotle, Organon.

[2] G. J. Klir, Fuzzy Set Theory, Prentice Hall Inc., 1997.

[3] Aristotle, Organon, Posterior Analytic.

[4] G. Leibniz, The Monadology, Sections 31, 32.

[5] Nassikas, A.A, Miminum Contradictions Everything, Reviewed by Duffy, M.C., Ed. Whitney, C.K., Hadronic Press, p. 185, ISBN: 1-57485-061-X, 2008.

[6] Nassikas, A. A., "Basic Statements Required for a Minimum Contradictions in Physics, Review Bulletin of Calcutta Mathematical Society," vol. 14, iss. 1, pp. 1-8, 2006.

[7] C. Hartonas, Basic Logic (in Greek), Ziti Publ., Thessaloniki, Greece, 2000.

[8] E. Nagel and J. R. Newman., Gödel's Proof, N.Y. University Press, New York, 1958.

[9] J. B. Rosser, "An Informal Exposition of Proofs of Gödel's Theorems and Church's Theorem," Journal of Symbolic Logic, vol. 4, 1939.

[10] H. Putnam, Book Review: Shadows of the Mind by R. Penrose. Bulletin of the American Mathematical Society, 1995.

[11] R. Penrose, Shadows of the Mind, Oxford University Press, 1994.

[12] B. Mandelbrot, The Fractal Geometry of Nature, Freeman. N. Y., 1977.

[13] Francis Jeffry Pelletier, "Review of Metamathematics of fuzzy logics in The Bulletin of Symbolic Logic," vol. 6, iss. 3, pp. 342-346, 2000.

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From 1990 to 1995, he wrote the book "The Claim for Minimum Contradictions" published by Trohalia in Greek (220 pages, ISBN: 960-7022-64-5). Starting point for writing the book were basic philosophical matters related with the deterministic or in-deterministic perception and the existence of Free Will, the ongoing interest around the Gödel Theorem and Fuzzy Logic, the basic issue of physics that is the unification of general relativity with quantum mechanics and the growing indication of Zero Point Energy very related to his subject i.e. to Renewable-New Forms of Energy. This book was the basis for all his later work. In 2008, he wrote the book "Minimum Contradictions Everything" (based on his 1995 book, his published papers and on a theorem he stated in 2004), Reviewed and Edited by M.C. Duffy and C.K. Whitney, published by Hadronic Press U.S. and distributed by Amazon (185 pages, ISBN: 1-57485-061-X). He holds 4 patents with 2 of them having been applied. E-mail: a.a.nass@teilar.gr